Mathematics Assignment For Class XII

General Directions For Students :Whatever be the notes provided , everything must be copied in the maths copy and then do the homework in the same copy

Chapter 2: Inverse Trigonometric Function (Part -2)

Topic : Finding the principal value (continued), Properties(First four properties) of Inverse Trigonometric Function

Property 1.

(i) $\sin(\sin^{-1} x) = x, \ |x| \le 1$ (ii) $\cos(\cos^{-1} x) = x, |x| \le 1$

(iii) $\tan(\tan^{-1} x) = x, x \in R$ (iv) $\cot(\cot^{-1} x) = x, x \in R$

(iv) $\sec(\sec^{-1} x) = x, |x| \ge 1$ (v) $\cos ec(\cos ec^{-1} x) = x, |x| \ge 1$

Property 2

(i) $\sin^{-1}(\sin x) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, (ii) $\cos^{-1}(\cos x) = x, x \in [0, \pi]$

(iii) $\tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (iv) $\cot^{-1}(\cot x) = x, x \in (0, \pi)$

(iv) $\sec^{-1}(\sec x) = x, \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ (v) $\cos ec^{-1}(\cos ecx) = x, \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Property 3

(i) $\sin^{-1}(-x) = -\sin^{-1} x, x \in |x| \le 1$ (ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in |x| \le 1$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x, x \in R$ (iii) $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$

(iv) $\cos ec^{-1}(-x) = -\cos ec^{-1} x, x \in |x| \ge 1$ (v) $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \ge 1$

Property 4

(i) $\cos ec^{-1} x = \sin^{-1}\left(\frac{1}{x}\right), |x| \ge 1$ **(** $\cot^{-1} x = \begin{cases} \tan^{-1} x, & x > 0 \\ \pi + \tan^{-1}\frac{1}{x}, & x < 0 \end{cases}$ **iii)**

(ii) $sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right), |x| \ge 1$

Exercise 2.2 Q.1.(i) Evaluate $\sin^{-1}(\sin 2)$

since $2 \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\sin^{-1}(\sin 2) \neq 2$

We know that $\pi - 2 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin(\pi - 2) = \sin 2$

$$\therefore \sin^{-1}\left(\sin(\pi - 2)\right) = \pi - 2$$

Exercise 2.2 Q2(i) Prove $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x, x < 1$

let x = tan y since x <1, tany<1

$\Rightarrow$0<y<$\frac{\pi}{4}$ $\Rightarrow \frac{\pi}{4} < \frac{\pi}{4}$ +y<$\frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\left(\frac{1+x}{1-x}\right) = \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan y}{1 + \tan\frac{\pi}{4}, \tan y}\right) = \tan^{-1}\left(\tan(\frac{\pi}{4} + y)\right)$$

$$\Rightarrow \tan^{-1}\left(\tan(\frac{\pi}{4} + y)\right) = \frac{\pi}{4} + y$$

$$\Rightarrow \frac{\pi}{4} + y = \frac{\pi}{4} + \tan^{-1} x \qquad \because y = \tan^{-1} x$$

Exercise 2.2 Q.4.(i) Write the following function in simplest form $\tan^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)$

$$\tan^{-1}\left(\frac{1-\cos x}{1+\cos x}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Exercise 2.2 Q.4.(iii) $\tan^{-1}\left(\frac{2\sqrt{x}}{1-x^2}\right)$

Let $\sqrt{x} = \tan\theta$

Then $\tan^{-1}\left(\frac{2\sqrt{x}}{1-x^2}\right) = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) \Rightarrow \tan^{-1}\left(\tan 2\theta\right) = 2\theta$

$\Rightarrow 2\tan^{-1}\left(\sqrt{x}\right) \qquad \because \theta = \tan^{-1}\sqrt{x}$

Homework:

Exercise 2.1 Q17(ii), Q18(ii)

Exercise 2.2 Q1 (iii),(v), Q3(ii),Q4 (ii),(iv),(viii)